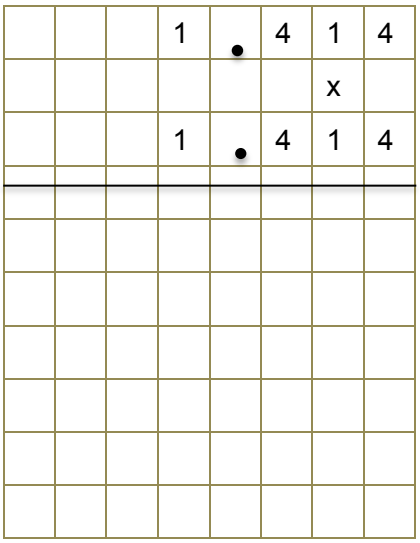
Squaring square roots

Well, first I used my little brother's calculator and found that the $\sqrt{2} = 1.414$

Then my teacher said that if I squared the square root of 2 on my calculator that I would get 2 because squaring is the opposite operation from square rooting. So, on his calculator I squared my answer, 1.414, and yes, on the calculator, it came out 2. But I think that couldn't be correct.

1. How do you think that I knew that 1.414 x 1.414 couldn't be 2? Please explain.

2. OK. Please show me what happens with some long multiplication.



On my own calculator $\sqrt{2} = 1.414213562$. So, maybe I have to be more precise than using my little brother's calculator. When I square that answer, 1.414213562, on my calculator, I get 2 again. Good? But when I do long multiplication, I still get something weird.

3. Why is that?

									1	.	4	1	4	2	1	3	5	6	2
								x											
									1	.	4	1	4	2	1	3	5	6	2
									2	8	2	8	4	2	7	1	2	4	
								8	4	8	5	2	8	1	3	7	2	-	
							7	0	7	1	0	6	7	8	1	0	-	-	
						4	2	4	2	6	4	0	6	8	6	-	-	-	
					1	4	1	4	2	1	3	5	6	2	-	-	-	-	
				2	8	2	8	4	2	7	1	2	4	-	-	-	-	-	
			5	6	5	6	8	5	4	2	4	8	-	-	-	-	-	-	
		1	4	1	4	2	1	3	5	6	2	-	-	-	-	-	-	-	
	5	6	5	6	8	5	4	2	4	8	-	-	-	-	-	-	-	-	
1	4	1	4	2	1	3	5	6	2	-	-	-	-	-	-	-	-	-	
1	.	9	9	9	9	9	9	9	9	8	9	4	4	7	2	7	8	4	4

4. Why didn't the product of the $\sqrt{2}$ and $\sqrt{2} = 2\ exactly$? Please explain your thoughts here.