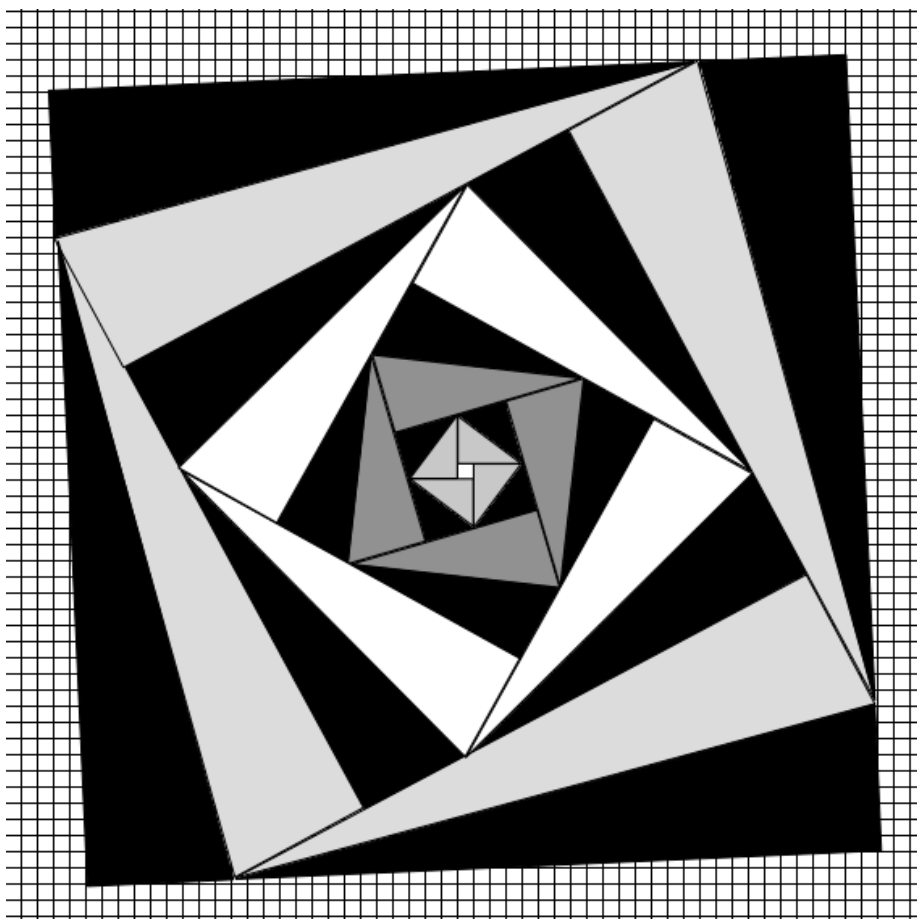
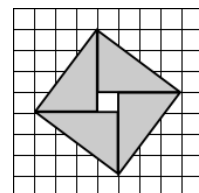


Spiraling Triples

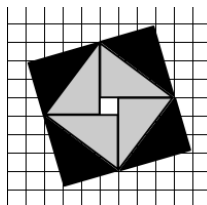


This is a picture of sets of Pythagorean Triple right triangles carefully nested together. The innermost triangles are 3-4-5 triangles. (For units, I'm just counting little squares.) Those first 3-4-5 triangles form a square. In the center part of those 4 nested triangles is also a little, 1 x 1 square where the triangles don't overlap.

1. What is that first square's (not the little 1 x 1 square's) side length?



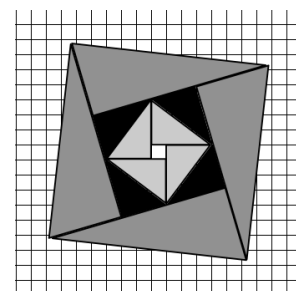
To make the second square, I reflected each of those initial triangles over their hypotenuses (hypotenuses?) and colored that second round of triangles black.



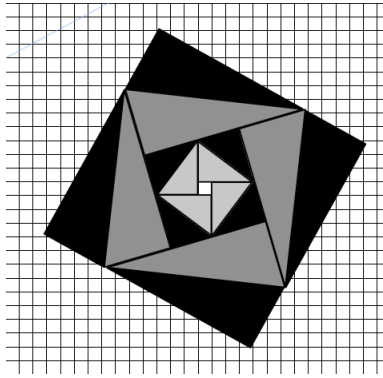
2. What is the measure of the black square's side? Please explain how you know that.

For the third round of triangles, I added 5-12-13 triangles.

3. What is the dimension of this next gray square?



I then reflected those gray triangles again, over their hypotenuses, to get another round of triangles and a new black square.

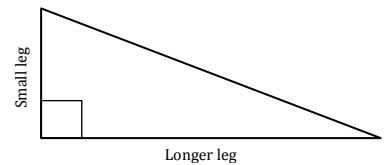


4. You can figure out that the next round of triangles will have a leg that is what length?
5. Will that leg (the leg that will be adjacent to the black triangle) be the new right triangle's short or long leg?

Before I ask you to explain more about these triangles and squares, let's look at some patterns that are relevant to this investigation.

1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144

6. Describe at least two patterns that you see in the numbers to the left.



On the right you see a table of some Pythagorean triples side lengths for right triangles. Notice that their small legs have lengths of consecutive odd numbers.

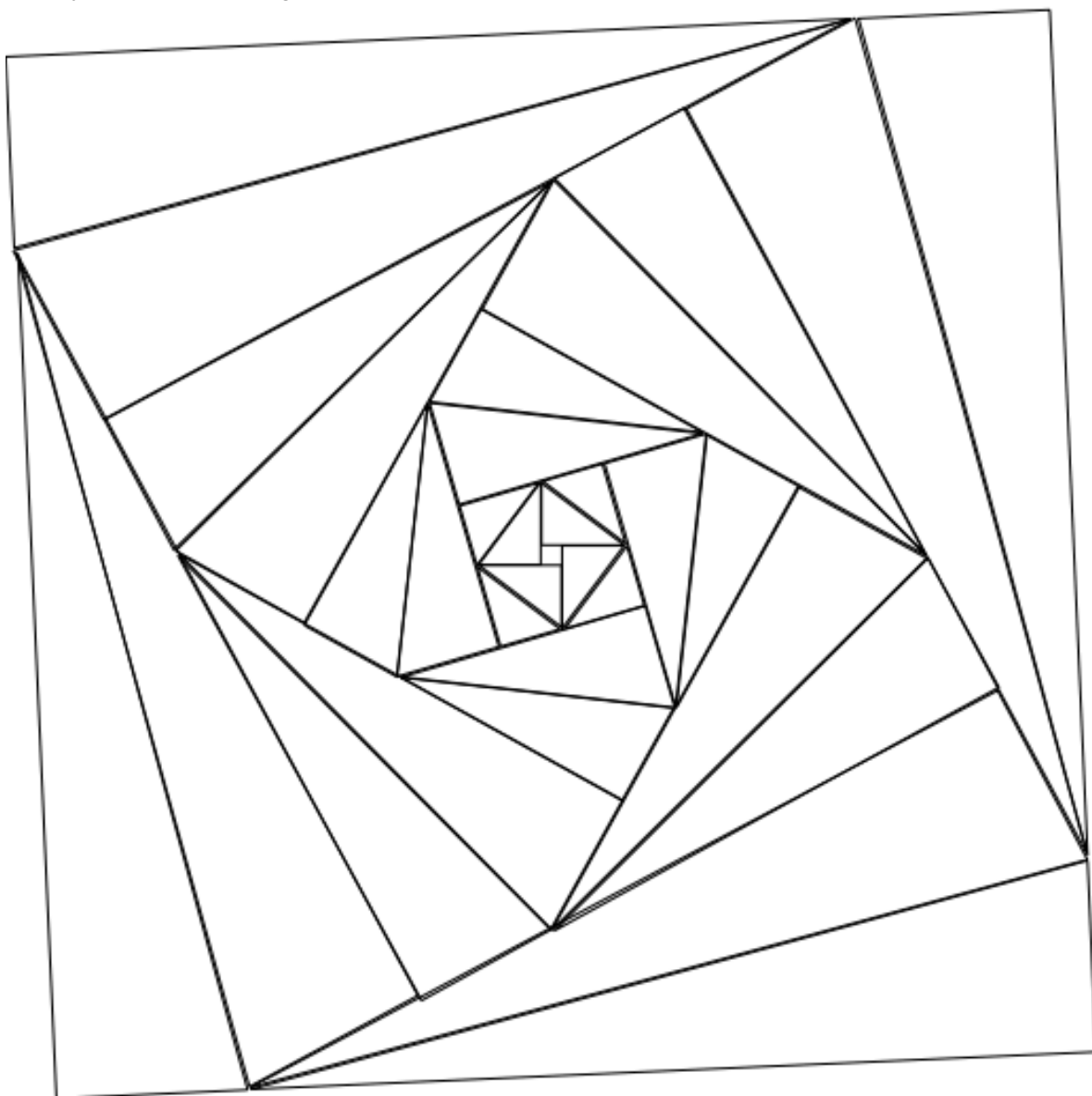
Small leg length	Long leg length	Hypotenuse
3	4	5
5	12	13
7	24	25
9	40	41
11	60	61
13	84	85
15	112	113

7. Can you deduce any patterns for the long leg lengths or hypotenuse numbers? Please explain.
8. What do you think that the next row to this table will be?

Now for my final question about the Spiraling Squares picture:

9. Why do you think that this image look like spiraling squares?

10. Color your own Spiraling Triple.



Source of inspiration: Diane Venters and Elaine Krajenke Ellison of *Mathematical Quilts - No Sewing Required*

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