Approximations for Pi

Any of these approximations for \( \pi \) could be used to involve students in the history of mathematics, to review long division, to practice order of operations in calculator work, or just for fun.

Of course the amazing fact that there is always the same ratio of any circle’s diameter to it’s diameter should be the beginning of any \( \pi \) work, these are all interesting and fun methods for finding approximations of \( \pi \).

\( \frac{22}{7} \) is often used as an approximation for \( \pi \). However, \( \pi \) is an irrational number. That means that it can’t be expressed as a fraction composed of an integer numerator and denominator. With calculus and computers, \( \pi \) has been calculated to 10 trillion digits. Below are the first digits of \( \pi \).

\[ \pi \approx 3.141592653589793238462643383279502884197169399375105820974944592307. \]

1. Use long division to calculate \( \frac{22}{7} \) as a decimal number and compare that number to the string of digits that are shown above. Is \( \frac{22}{7} \) bigger or smaller than \( \pi \)? How do you know?

2. Using long division again, calculate \( \frac{333}{106} \) as a decimal number and compare that number to the string of digits is shown above. Is \( \frac{333}{106} \) bigger or smaller than \( \pi \)? How do you know?

3. Using long division one more time, calculate \( \frac{355}{113} \) as a decimal number and compare that number to the string of digits is shown above. Is \( \frac{355}{113} \) bigger or smaller than \( \pi \)? How do you know?

4. \( \sqrt{2} + \sqrt{3} \) is also used as an approximation of \( \pi \). Calculate these values using your calculator. \( \sqrt{2} \) and \( \sqrt{3} \) are also irrational numbers. So the approximations that you get for those two values of this formula are truncated as well. To how many digits do you think this approximation of \( \pi \) is accurate?
5. How accurate is this approximation of $\pi$? 
\[
3 + \frac{8}{60} + \frac{29}{60^2} + \frac{44}{60^3}
\]

6. Using your calculator figure out how accurate this approximation for $\pi$ is? How many digits of $\pi$ seem to be consistent to the above given value? 
\[
\sqrt{7 + \sqrt{6 + \sqrt{5}}}
\]

7. The famous Indian mathematician, Ramanujan (1887 – 1920), devised this approximation for $\pi$.
\[
\frac{9}{5} + \frac{9}{\sqrt{5}}
\]
To how many digits is this computation accurate?

8. How accurate is this one? 
\[
\frac{7^7}{4^9}
\]

9. Last one. How about this approximation. To how many digits is this approximation accurate?
\[
\left(3^4 + 2^4 + \frac{1}{2 + \left(\frac{2}{3}\right)^2}\right)^{\frac{1}{3}}
\]


Brought to you by [Yummymath.com](http://www.yummymath.com)